Limits, Noise, and Research

Name: _____ Date: _____

1. Count each error neighborhood EASY

For an n-bit protected message, the receiver may see the original or any one of n single-bit flips. Each correctable message therefore needs $n + 1$ exclusive received strings.

n	All strings 2^n	Slots per message $n + 1$	$16(n + 1)$ needed	Can 16 fit?
5	32	6	_____	
6	64	7	_____	
7	128	8	_____	

Why can no clever rearrangement make five or six bits sufficient?

What is special about $16(7 + 1) = 2^7$?

2. Derive the bound CHALLENGE

Write a general inequality for M protected messages of length n that must correct every single-bit error.

Use it to test whether 20 messages can be protected with 7 bits. Show your calculation and conclusion.

Find the smallest n not ruled out by this counting bound for 32 messages.

3. Transmission cost EASY

Hamming(7,4) converts every four data bits into seven transmitted bits.

a) Extra bits per four-bit message: _____
b) Transmitted bits for one 8-bit byte: _____
c) The complete text used in the slides has 3,359,405 bytes. Confirm that encoding it uses 47,031,670 transmitted bits. Show the multiplication.

d) Compare with repeating every bit three times. How many transmitted bits would that require?

4. What does a 3 percent flip rate mean? EASY

Circle the correct interpretation.

a) Exactly 3 bits flip in every 100. Each transmitted bit independently has probability 0.03 of flipping.
b) Every seven-bit block gets one error. Some blocks get 0, 1, 2, or more errors.

Why can recovery be high but not perfect at a 3 percent flip rate?

5. Predict recovery with probability CHALLENGE

For independent flip probability p, a seven-bit codeword has no errors with probability $(1-p)^7$ and exactly one error with probability $7p(1-p)^6$.

1. Write the probability that Hamming(7,4) is within its guarantee:

$$P(0 \text{ or } 1) = \_\_\_\_\_$$

2. Evaluate it for $p = 0.03$. _____
3. A byte uses two codewords. Estimate the probability that both are within the guarantee by squaring your answer. _____
4. Repeat for $p = 0.10$. What changes most sharply?

_____

_____

6. Random errors versus bursts EASY

Match the channel to the description.

Independent noise	
Burst error	
Erasure	
Insertion/deletion	

Descriptions: a known position is unreadable; nearby bits fail together; each bit flips separately; bits are added or lost and positions shift.

Why might interleaving help against a burst?

_____

_____

7. Design a research test CHALLENGE

Choose one question.

- How does exact text recovery change as the flip rate moves from 0 to 20 percent?
- Can a different seven-bit code protect more than sixteen four-bit reports?
- How should a code change when two errors must be corrected?
- How does interleaving change damage from a fixed-length burst?

Claim or prediction:

_____

Variables and test procedure:

_____

_____

Evidence to record:

_____

What result would change your mind?

_____

Reflection EASY

The strongest guarantee I can state precisely is: _____

Original NILA Mission Codebook

The sixteen data messages protected by the lesson's Hamming code.

Code	Mission report	Code	Mission report
0000	Systems normal	1000	Liquid water detected
0001	Battery low	1001	Organic molecules detected
0010	Camera ready	1010	Possible biological pattern
0011	Camera damaged	1011	Mission emergency
0100	Drill ready	1100	Radiation spike
0101	Drill jammed	1101	Surface crack opening
0110	Ice sample collected	1110	Warm plume detected
0111	Salt detected	1111	Return to shelter

Key bound: if M length-n codewords must correct every one-bit error, their radius-one neighborhoods cannot overlap, so $M(n+1) \leq 2^n$. Equality for $M = 16, n = 7$ is why Hamming(7,4) is called a perfect code.