

Discrete Structures

Fight for Survival

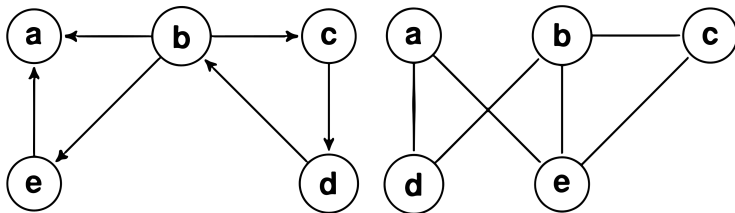
Dr. Mudassir Shabbir

August 27, 2026



Connectivity in Graphs

A graph $G = (V, E)$ is a **connected** if, for each pair of vertices, there exists at least one path that joins them.

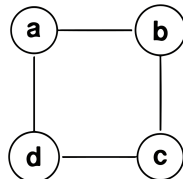


Connectivity in Undirected Graphs

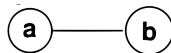
An undirected graph is called **connected**, if there is a path between every pair of vertices in the graph.

An undirected graph that is not connected is called **disconnected**.

We say that we disconnect a graph when we remove vertices or edges, or both, to produce a disconnected subgraph.



Connected

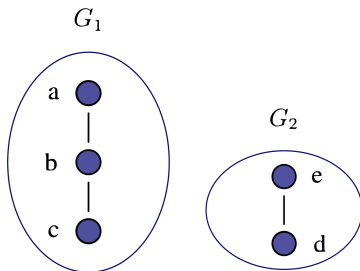


Disconnected

Connected Components

A connected component of a graph G is a maximal connected subgraph of G .

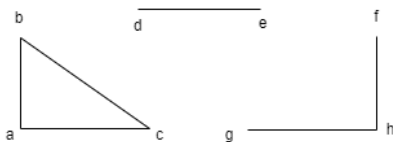
A graph G that is not connected has two or more connected components that are disjoint and have G as their union.



G_1 and G_2 are two Connected Components of graph G .

Connected Components

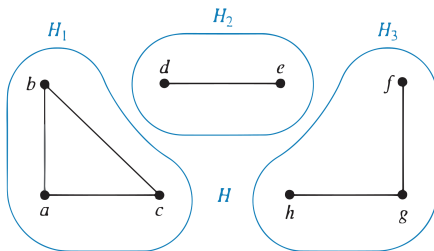
What are the connected components of the graph H shown in Figure?



Graph H

Connected Components

What are the connected components of the graph H shown in Figure?

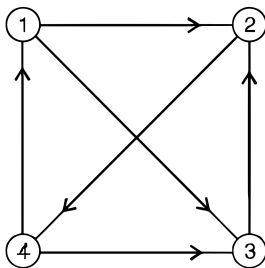


Solution: The graph H is the union of three disjoint connected subgraphs H_1 , H_2 , and H_3 , shown in Figure. These three subgraphs are the connected components of H .

Connectivity in directed Graphs

There are **two notions of Connectivity** in directed graphs, depending on whether the **directions of the edges** are considered.

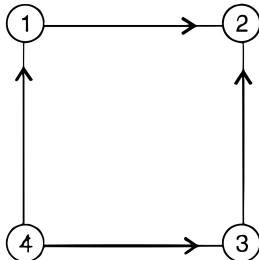
1. Strongly Connected Directed Graphs: A directed graph is strongly connected if there is a path from a to b and from b to a whenever a and b are vertices in the graph.



Connectivity in directed Graphs

There are **two notions of Connectivity** in directed graphs, depending on whether the **directions of the edges** are considered.

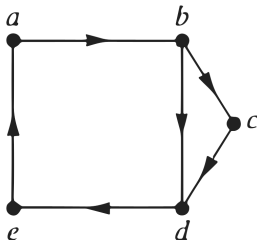
2. Weakly Connected: A directed graph is weakly connected if there is a path between every two vertices in the underlying undirected graph. .



Connectivity in directed Graphs

Practice Question:

Is the directed graphs G in Figure strongly connected or weakly connected?

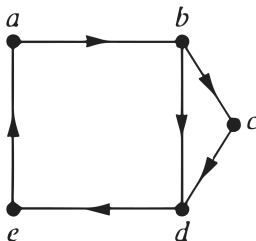


G

Connectivity in directed Graphs

Practice Question:

Is the directed graphs G in Figure strongly connected or weakly connected?



G

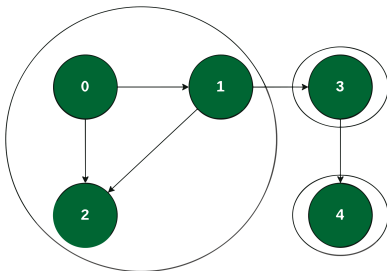
Solution: G is strongly connected because there is a path between any two vertices in this directed graph.



Connected Components

The maximal strongly connected subgraphs, are called the strongly connected components or strong components of G .

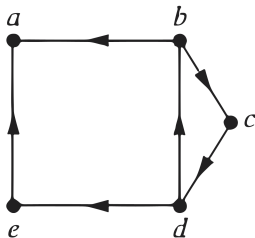
Note that if a and b are two vertices in a directed graph, their strong components are either the same or disjoint.



This graph has **three** strongly connected components.

Connected Components

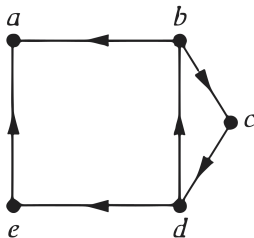
How many strongly connected components are there in graph H ?



H

Connected Components

How many strongly connected components are there in graph H ?



H

Solution: The graph H has **three strongly connected components**, consisting of the vertex a ; the vertex e ; and the subgraph consisting of the vertices b , c , and d and edges (b, c) , (c, d) , and (d, b) .