Discrete Structures

Fight for Survival

Dr. Mudassir Shabbir

August 27, 2026

Conditional Propositions

Conditional Proposition : $p \to q$

Read as: If p then q

p	q	$p \to q$

Example:

If x is a positive even number, then x is divisible by 2.
If x is not a positive even number, then x may or may not be divisible by 2.

Note: A conditional proposition is false only when the premise is true and the conclusion is false.

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Conditional Proposition : $p \rightarrow q$

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Read as: If p then q

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True	False	
False	True	
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Trivially True Conditionals

Trivially True Conditional:

If the conclusion is true regardless of the hypothesis part, the conditional statement is trivially true.

Example:

If elephants can fly, then 2 + 2 = 4.

Truth Table:

Hypothesis	Conclusion	Conditiona
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If elephants can fly, then 2 + 2 = 4.

Explanation:

- Since the conclusion (2 + 2 = 4) is always true,
- it doesn't matter whether elephants can fly or not.

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Explanation:

- Since the conclusion (2 + 2 = 4) is always true,
- it doesn't matter whether elephants can fly or not.
- This makes the conditional statement trivially true.

Vacuously True Conditionals

Vacuously True Conditional:

If the hypothesis is false, then the conditional statement is vacuously true regardless of the conclusion part.

Example:

If unicorns exist, then 1 + 1 = 3.

Truth Table:

Hypothesis	Conclusion	Conditiona
False	True	True
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Explanation:

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- the conditional statement is automatically considered true,

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Equivalence of Conditional Statements

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- These two statements have different meanings and can have different truth values.

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- These two statements have different meanings and can have different truth values.

Example:

Let p be "It's raining" and q be "I carry an umbrella."

$p \to q$: If it's raining, I carry an umbrella.

$q \to p$: If I carry an umbrella, it's raining.

Equivalence of Compound Propositions

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Equivalence of Compound Propositions

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Is the truth value of $(p \to r) \land (q \to r)$ always the same as the truth value of $(p \lor q) \to r$?

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Yes, the truth value of $(p \to r) \land (q \to r)$ is always the same as the truth value of $(p \lor q) \to r$.

Explanation:

- Let's examine both propositions with a truth table.

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- If both $(p \to r)$ and $(q \to r)$ are true, it means p and q imply r.

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- Let's examine both propositions with a truth table.
- If both $(p \to r)$ and $(q \to r)$ are true, it means p and q imply r.
- In $(p \vee q) \to r$, the condition $p \vee q$ also implies r.

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- Let's examine both propositions with a truth table.
- If both $(p \to r)$ and $(q \to r)$ are true, it means p and q imply r.
- In $(p \lor q) \to r$, the condition $p \lor q$ also implies r.
- Therefore, the two propositions have matching truth values.

Equivalence of Compound Propositions: Truth Table

Truth Table for $(p \to r) \land (q \to r)$ and $(p \lor q) \to r$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \land (q \to r)$	$(p \lor q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Equivalence of Different Statements

Statement	Equivalent Statement
$p \to q$	p is sufficient for q
$p \to q$	q is necessary for p
$p \to q$	p only if q

Explanation:

- All four statements describe the same implication relationship between p and q.
- The statements emphasize different aspects of the implication, but they are logically equivalent.

Sufficient Condition

Statement: If a sufficient condition is true, then it is guaranteed that the conclusion holds.

Example:

Suppose we have an exam with the following rule:

- If you score at least 80% on the midterm, then you will pass the course.

Here, scoring 80% or higher on the midterm is a sufficient condition for passing the course.

Explanation:

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- If a student fulfills the sufficient condition of scoring at least 80%, they are guaranteed to pass.

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Explanation:

- If a student fulfills the sufficient condition of scoring at least 80%, they are guaranteed to pass.
- The exam rule ensures that achieving the sufficient condition automatically leads to the desired conclusion.

Sufficient Condition and False Hypothesis

Statement: If the sufficient condition (hypothesis) is false, we can't say anything about the conclusion.

Example:

Consider the rule for getting a bonus at work:

- If you work overtime on weekends, then you'll receive a bonus.

Here, working overtime on weekends is a sufficient condition for receiving a bonus.

Explanation:

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Consider the rule for getting a bonus at work:

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Explanation:

- If an employee doesn't work overtime on weekends, we can't determine whether they'll receive a bonus or not.

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- If an employee doesn't work overtime on weekends, we can't determine whether they'll receive a bonus or not.
- The rule only guarantees a bonus if the sufficient condition (working overtime) is met.

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Explanation:

- If an employee doesn't work overtime on weekends, we can't determine whether they'll receive a bonus or not.
- The rule only guarantees a bonus if the sufficient condition (working overtime) is met.
- If the sufficient condition is not met (i.e., it is false), the conclusion (receiving a bonus) is undefined.

Necessary Condition

Statement: A necessary condition must be true for the conclusion to hold.

Example:

Consider the requirement for attending a concert:

- To enter the concert, you must show a valid ticket.

Here, having a valid ticket is a necessary condition to enter the concert.

Explanation:

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Here, having a valid ticket is a necessary condition to enter the concert.

Explanation:

- If you don't have a valid ticket, you cannot enter the concert.
- The condition is necessary for accessing the event; it's a prerequisite.

Necessary Condition and False Conclusion

Statement: If the necessary condition is false, we can conclude that the conclusion cannot hold.

Example:

Suppose you have a password-protected app with the following rule:

- To access the app, you must know the correct password.

Here, knowing the correct password is a necessary condition to access the app.

Explanation:

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Suppose you have a password-protected app with the following rule:

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- If you don't know the correct password, you can't access the app; it's a must-have.

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- To access the app, you must know the correct password.

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Explanation:

- If you don't know the correct password, you can't access the app; it's a must-have.
- If the necessary condition is false, we can conclude that the conclusion is impossible.

Converse, Contrapositive, and Inverse

Original Statement: If a person is a programmer, then they know how to code.

Converse	Contrapositive	Inverse
If a person knows how to code, then they are a programmer.	If a person doesn't know how to code, then they are not a programmer.	If a person is not a programmer, then they don't know how to code.

Explanation:

Converse, Contrapositive, and Inverse

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Explanation:

- The converse switches the hypothesis and conclusion of the original statement.

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- The converse switches the hypothesis and conclusion of the original statement.
- The inverse negates both the hypothesis and conclusion of the original statement.

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Explanation:

- The converse switches the hypothesis and conclusion of the original statement.
- The inverse negates both the hypothesis and conclusion of the original statement.
- The contrapositive negates both the hypothesis and conclusion of the original statement and switches them.

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- The converse switches the hypothesis and conclusion of the original statement.
- The inverse negates both the hypothesis and conclusion of the original statement.
- The contrapositive negates both the hypothesis and conclusion of the original statement and switches them.
- These variations may or may not have the same truth values as

Bi-conditional Propositions

Statement: *A bi-conditional proposition ($P \leftrightarrow Q$) is true if both parts have the same truth value.*

Example:

Consider a rule for a two-way ticket to a theme park:

- You can use the two-way ticket if and only if you enter the park in the morning.

Here, using the two-way ticket is bi-conditional to entering the park in the morning.

Truth Table:

P	Q	$P \leftrightarrow Q$
True	True	True
True	False	False
False	True	False
False	False	True

Examples of Biconditional Propositions

Example 1:

- February has 29 days if and only if it's a leap year.

Example 2:

- One camera is necessary and sufficient to completely monitor any convex area.

Example 3:

- You'll pass the exam if and only if you've studied for at least $10,000$ minutes.

Funny Example:

- You'll become a master chef if and only if you can microwave popcorn without burning it.

Logically Equivalent Propositions

Propositions are Logically Equivalent:

Two propositions are logically equivalent when they have the same truth values for every possible combination of truth values of their variables.

Example:

Consider propositions P : It's raining and Q : I need an umbrella. They are logically equivalent because whenever it's raining, I need an umbrella, and whenever I don't need an umbrella, it's not raining.

Equivalence of $p \to q$ and $\neg q \to \neg p$

Truth Table for $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for $\neg q \to \neg p$

q	$\neg q$	$\neg q \to \neg p$
p	q	$\neg q \to \neg p$
T	T	T
T	F	F
F	T	T
F	F	T

Conclusion: Yes, $p \to q$ is equivalent to $\neg q \to \neg p$. They have the same truth values for all combinations of p and q.

Are Converse and Inverse Propositions Equivalent?

Question: Is it true that converse propositions are equivalent to inverse propositions?

Equivalence: $\neg(p \to q)$ and $p \land \neg q$

Propositions: $\neg(p \to q)$ and $p \land \neg q$

Truth Table (Step 1):

p	q	$p \land \neg q$
T	T	F
T	F	T
F	T	F
F	F	F

Equivalence: $\neg(p \to q)$ and $p \land \neg q$

Propositions: $\neg(p \to q)$ and $p \land \neg q$

Truth Table (Step 1):

p	q	$p \land \neg q$
T	T	F
T	F	T
F	T	F
F	F	F

Truth Table (Step 2):

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Equivalence: $\neg(p \to q)$ and $p \land \neg q$

Propositions: $\neg(p \to q)$ and $p \land \neg q$

Truth Table (Step 1):

p	q	$p \land \neg q$
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F	F	F

Truth Table (Step 2):

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Conclusion (Step 3): $\neg(p \to q)$ is equivalent to $p \land \neg q$ as they have the same truth values for all combinations of p and q.

Tautology: Definition and Example

Tautology: A proposition that is always true, regardless of the truth values of its variables.
Example: $p \lor \neg p$ is a tautology.
Truth Table:

p	$\neg p$	$p \lor \neg p$
T	F	T
F	T	T

Contradiction: Definition and Example

Contradiction: A proposition that is always false, regardless of the truth values of its variables.

Example: $p \wedge \neg p$ is a contradiction.

Truth Table:

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Tautology and Contradiction in Logical Operations

Question: If t is a tautology and c is a contradiction, then what are the values of $p \wedge t$ and $p \wedge c$?

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Question: If t is a tautology and c is a contradiction, then what are the values of $p \wedge t$ and $p \wedge c$?

Answer:

- $p \wedge t$ is equivalent to p for any value of p since t is always true.
- $p \wedge c$ is False for any value of p since c is always false.