Discrete Structures

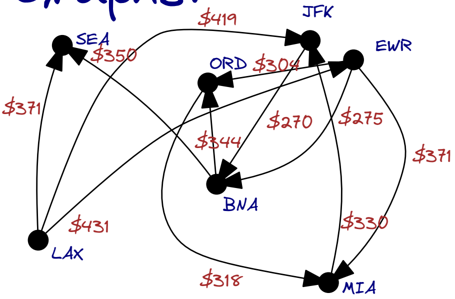
Fight for Survival

Dr. Mudassir Shabbir

August 27, 2026

Weighted Graphs?

Graph can also represent weighted networks with weights on edges.



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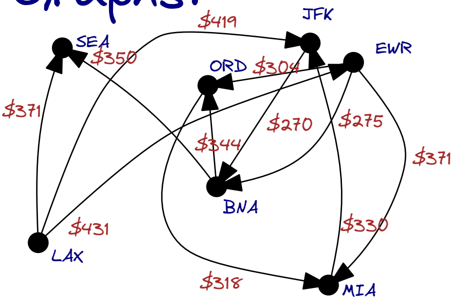


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EWR -> MIA 371, BNA 275, ORD 304
JFK -> BNA 270
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```

Weighted Adjacency List

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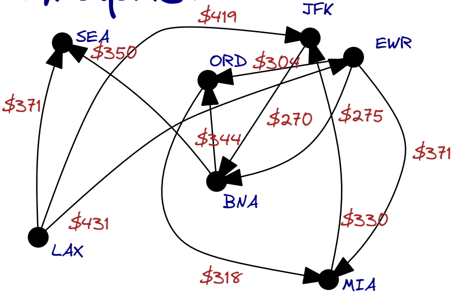

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Weighted Adjacency List

Weighted Adjacency Matrix

	EWR	JFK	MIA	BNA	ORD	SEA	LAX
EWR	0	0	371	275	304	0	0
JFK	0	0	0	270	0	0	0
MIA	0	330	0	0	0	0	0
BNA	0	0	0	0	344	250	0
ORD	0	0	318	0	0	0	0
SEA	0	0	0	0	0	0	0
LAX	431	419	0	0	0	371	0

Weighted Graphs?

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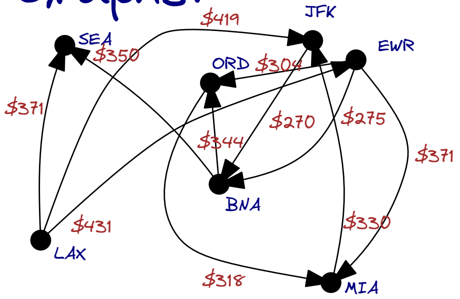


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Weighted Adjacency Matrix

A weighted graph can be directed or undirected!

	EWR	JFK	MIA	BNA	ORD	SEA	LAX
EWR	0	0	371	275	304	0	0
JFK	0	0	0	270	0	0	0
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BNA	0	0	0	0	344	250	0
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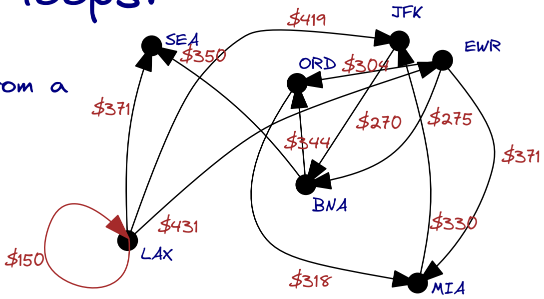
Weighted Adjacency List

Self-loops?

A self-loop is an edge from a vertex to itself.



Example: a plane that takes off from LAX, wanders around for a while and then lands back at LAX.

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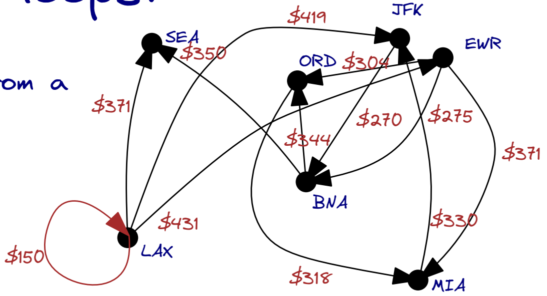

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ORD -> MIA 318
SEA ->
LAX -> SEA 371, JFK 419, EWR 431, LAX 150

Example: a plane that takes off from LAX, wanders around for a while and then lands back at LAX.

	EWR	JFK	MIA	BNA	ORD	SEA	LAX
EWR	0	0	371	275	304	0	0
JFK	0	0	0	270	0	0	0
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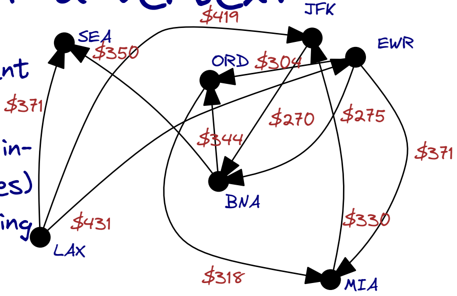
BNA	0	0	0	0	344	250	0
ORD	0	0	318	0	0	0	0
SEA	0	0	0	0	0	0	0
LAX	431	419	0	0	0	371	150

DEGREE of a vertex?

is the number of edges adjacent to it.

For directed graphs, we have in-degree (number of incoming edges) and out-degree (number of outgoing edges).



Example: $deg^{-}(\text{SEA}) = 2, deg^{+}(\text{SEA}) = 0$

EWR -> MIA 371, BNA 275, ORD 304
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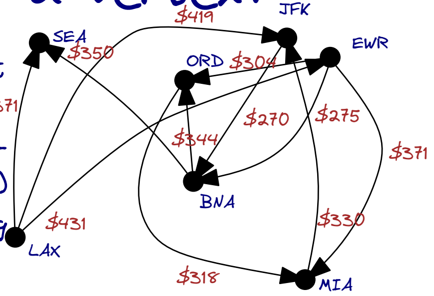

Example: $deg^{-}(\text{SEA}) = 2, deg^{+}(\text{SEA}) = 0$

ORD is an OUT-neighbor of BNA

BNA is an IN-neighbor of ORD

ORD is adjacent to BNA.

EWR -> MIA 371, BNA 275, ORD 304
JFK -> BNA 270
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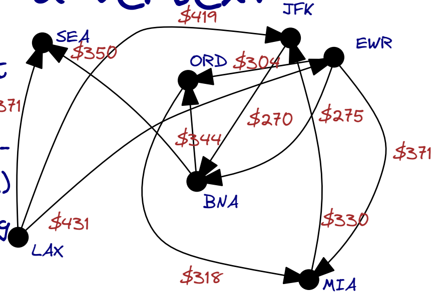

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Example: $deg^{-}(SEA) = 2, deg^{+}(SEA) = 0$

ORD is an OUT-neighbor of BNA

BNA is an IN-neighbor of ORD

ORD is adjacent to BNA.

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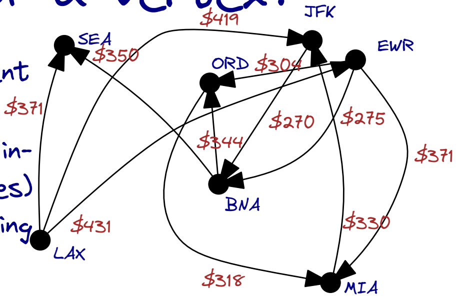

Example: $deg^-(\text{SEA}) = 2, deg^+(\text{SEA}) = 0$

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BNA is an IN-neighbor of ORD

ORD is adjacent to BNA.

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JFK -> BNA 270
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Question: Create a graph with the following degrees: $2, 2, 1, 3, 1.$

DEGREE of a vertex?

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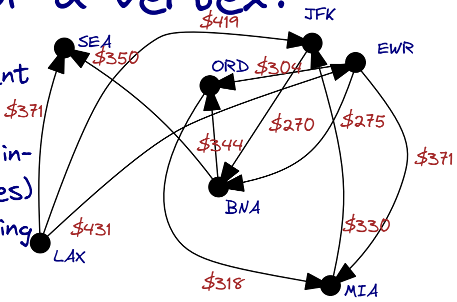

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ORD is an OUT-neighbor of BNA

BNA is an IN-neighbor of ORD

ORD is adjacent to BNA.

Question: Create a graph with the following degrees: $2, 2, 1, 3, 1$.

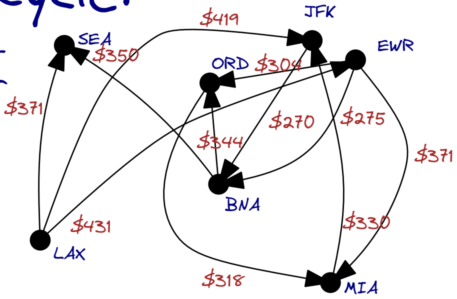
You can not! We will see why in a bit.

Path and Cycle?

A path is a sequence of vertices where each consecutive pair is adjacent.

Example: {BNA, ORD, MIA} is a path.

Example: {BNA, ORD, EWR, MIA} is NOT a path.



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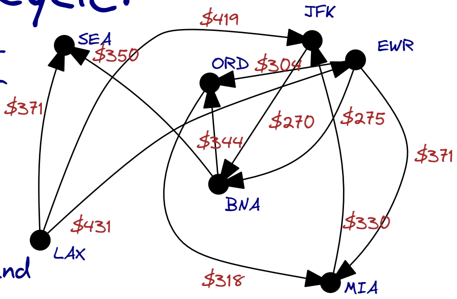
A cycle is a path that starts and ends at the same vertex.

Example: {BNA, ORD, MIA, EWR, BNA} is a cycle.



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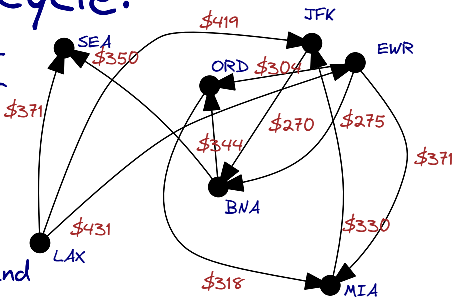

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A Direct Acyclic Graph (DAG) is directed graph that has no cycles.

Handshaking Lemma

- Lemma: The sum of degrees in an undirected graph is twice the number of edges. In particular, this must be even.

$$\sum_{v \in V} deg(v) = 2 \times |E|$$