

# Discrete Structures

## Fight for Survival

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# DISCRETE QUIZ

DIVIDE YOURSELVES INTO TEAMS OF SIZES 10, 11, OR 12. WE WILL PLAY A QUIZ GAME

- FIRST PRIZE: 5 POINTS TO TAKE!
- SECOND PRIZE: 5 POINTS TO GIVE!
- THIRD PRIZE: 5 MINUTES EXTRA ON FINAL EXAM!
- 10 POINTS FOR A CORRECT ANSWER, -5 FOR INCORRECT ANSWER.
- IF A QUESTION HAS MULTIPLE PARTS, YOU HAVE TO ANSWER ALL PARTS CORRECTLY, OTHERWISE YOUR ANSWER IS INCORRECT.
- YOU MAY PASS A QUESTION.
- 90 SECONDS FOR EACH QUESTION PER TEAM.
- AFTER HALF-TIME, YOU CAN BUY A PLAYER FOR 20 POINTS + A PLAYER!
- THERE ARE 29 QUESTIONS IN TOTAL.
- EXTERNAL HELP WILL RESULT IN A SERIOUS PENALTY FOR THE WHOLE TEAM.



# LET US BEGIN!



# Question 1

Let  $R$  and  $S$  be two relations both on  $A$ .

- 1 If  $R$  and  $S$  are both reflexive, is  $R \cup S$  also reflexive?
- 2 If  $R$  and  $S$  are both symmetric, is  $R \cup S$  also symmetric?
- 3 If  $R$  and  $S$  are both transitive, is  $R \cup S$  also transitive?

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YYN

## Question 2

Consider the following two statements: “100 percent of convicted felons consumed bread during their childhood.” and “People who consume bread as a child are likely to become felons.” Does the first statement logically imply the second?



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NO



## Question 3

In how many ways can you put 7 (different) pigeons into 8 pigeonholes if no two pigeons can be in the same pigeonhole?



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8!

## Question 4

What is the entry in **third column of sixth row** of Pascal's triangle?

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10

## Question 5

How many equivalence relations are there on  $A = \{1, 2, 3\}$ ?

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5

## Question 6

In a computer science class, there are 6 students. The teacher needs to assign them into project groups of 3. However, 2 students, Alice and Bob, refuse to work together on the same project. How many different valid group assignments are there?

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6

## Question 7

if  $X = \{1, 2, 3, 4, 5, 6, \{1\}\}$ . Find  $Y$ , and  $Z$  such that

$$Y = X \cup (X \cap \mathcal{P}(X))$$

and

$$Z = X \cap (X \cap \mathcal{P}(X))$$

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and

$$Z = X \cap (X \cap \mathcal{P}(X))$$

$$Y = X, Z = \{\{1\}\}$$

## Question 8

Prove that at any party there must be two people who have shaken hands with the **same number of others** present.

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Prove that at any party there must be two people who have shaken hands with the same number of others present.

Minimum number of handshakes is zero and maximum is  $n - 1$ . If there is someone who did not shake hands with anyone you can not have any who shook hands with everyone.

## Question 9

In a graph on 100 vertices, how many of the vertices can have **odd degree**?

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In a graph on 100 vertices, how many of the vertices can have **odd degree**?

All of them!

## Question 10

What is the maximum number of edges in a directed graph with **no self loops**?

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$$n^2 - n$$



## Question 11

If a graph can be disconnected by removing only one vertex, what is the maximum number of edges it can have?

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$$\binom{n-1}{2} + 1$$

## Question 12

Compare asymptotically, and give answers (which of the two functions grows faster) for all:

$$f_1(n) = (\log n)^a, \quad g_1(n) = n^b \text{ for } a, b > 0$$

and

$$f_2(n) = 2^{n \log n}, \quad g_2(n) = n!$$

and

$$f_3(n) = n^2, \quad g_3(n) = n(\log n)^{15}$$

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$$f_3(n) = n^2, \quad \gg g_3(n) = n(\log n)^{15}$$

## Question 13

True or False

- 1 If  $R$  is symmetric then  $\overline{R}$  is not symmetric.
- 2 If  $R$  is symmetric then  $\overline{R}$  is anti-symmetric.
- 3 If  $R$  is transitive then  $\overline{R}$  is not transitive.

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All lies!

## Question 14

What is the maximum number of trees on vertices  $V = \{1, 2, 3, 4\}$ ?

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16

## Question 15

There are 5 students applying to be part of a 4-person committee which consists of a president, a secretary, and two other members. How many different committees can be formed?



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60

## Question 16

What is the maximum number of edges in a directed graph (self-loops are allowed) that is **not strongly connected**?

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$$n^2 - n + 1$$

## Question 17

If a graph on  $n$  vertices can be disconnected by removing only one edge, what is the maximum number of edges it can have?



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$$\binom{n-1}{2} + 1$$

## Question 18

Let  $R$  and  $S$  be two relations both on  $A$ .

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YYY

## Question 19

True or False

$$\forall x, y \in \mathbb{Z}, \exists z \in \mathbb{Z} (x \leq z \leq y) \vee (x \geq z \geq y)$$

$$\forall x, y \in \mathbb{Z}, \exists z \in \mathbb{Z} (x < z \leq y) \vee (x > z \geq y)$$

## Question 19

True or False

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True and False



## Question 20

How many **nine digit strings** are there that contain exactly two 9's?

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$$\binom{9}{2} \times 10^7$$

## Question 21

Given a finite set  $X$ , with  $|X| = n$ , how many distinct symmetric relations on  $X$  exist?

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$$2^{\frac{n(n+1)}{2}}$$

## Question 22

What is the minimum number of edges required to make **Strongly Connected graph** on 100 vertices?

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100

## Question 23

How many five digit numbers are there that contain an 8 or a 9?



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How many five digit numbers are there that contain an 8 or a 9?

$$10^5 - 8^5$$

## Question 24

You are going on a trip and need to pack 4 shirts and 2 pairs of pants. If you **can choose any shirt and any pair of pants independently**, how many different outfit combinations are there?

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8

## Question 25

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and “People who do not consume bread as a child do not become felons.”  
Does the first statement logically imply the second?



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and “People who do not consume bread as a child do not become felons.”

Does the first statement logically imply the second?

YES

## Question 26

What is the maximum number of edges in a **directed graph** that has no **cycles**?

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What is the maximum number of edges in a directed graph that has no cycles?

$$\binom{n}{2}$$

## Question 27

You have a bookshelf with space for 8 books. You want to arrange 5 different math textbooks and 3 different novels on the shelf. If order matters (i.e., a specific book is in a specific slot), how many unique arrangements are there for these books?

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## Question 28

At a party, there are 20 guests. Everyone shakes hands with everyone else exactly once. How many total handshakes occur?

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190

## Question 29

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$n - 1$  and  $0$

THAT'S ALL FOLKS!

