Discrete Structures

Fight for Survival

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Drawing Conclusions from Given Information

Login involves inferring truth from given statements.

For example if:

1. One of them is a theif.
2. Exactly one of them is speaking the truth.

A: I am not the theif B: A is the theif C: I am not the theif

Who is the theif?

Drawing Conclusions from Given Information

Suppose A is the theif. Then both B and C are speaking the truth.

However,

2. Exactly one of them is speaking the truth.

A: I am not the theif B: A is the theif C: I am not the theif

Conclusion: A is not the theif.

Drawing Conclusions from Given Information

Suppose B is the theif. Then both A and C are speaking the truth.

However,

2. Exactly one of them is speaking the truth.

A: I am not the theif

B: A is the theif

C: I am not the theif

Conclusion: B is not the theif.

Drawing Conclusions from Given Information

Suppose C is the theif. Then only A is speaking the truth.

We know that A and B are not theives. So,

A: I am not the theif

B: A is the theif

C: I am not the theif

Conclusion: C is the theif.

Drawing Conclusions from Given Information

What if we have n persons, and exactly k of them are speaking the truth? Who is the thief?

Takeaways:

- We can infer new statements (conclusions) by carefully considering the given statements and premise.
- Things can get complex quickly, so we need to formalize and systemize our method of reasoning.

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- Relation between statements (structure)

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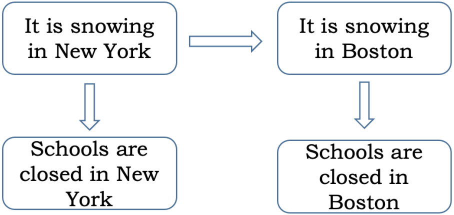

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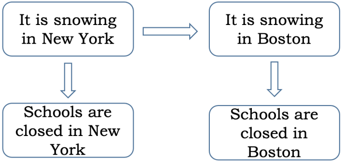
Formalize - How to Reason?



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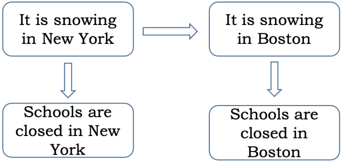

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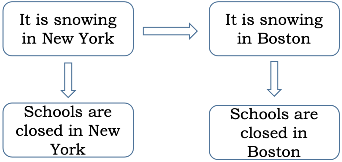

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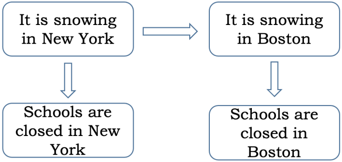

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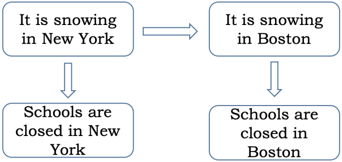

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Our goal will be to formalize the process of reasoning.

- What do we mean by statements (mathematically)?
- What can be a good way to capture relations between statements?
- How can we write new statements from known ones, such as by performing some operations on them?

Why - Need to Reason?

This formalization is key to

- Constructing precise mathematical arguments,
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Now we are going to formally start Chapter 1.

1.1 Propositions and Logical Operations

Proposition: A statement that is either true or false, but not both.

Statement	Proposition	Truth Value
29 is a prime number		
Open the door		
$x + y > 5$,		
Earth is the only planet where life exists		??
For every positive integer n, there is a prime number larger than n		

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Why we defined our basic building block this particular way?

- Simplest, concise, and the most un-ambiguous way of declaring a fact / information
- Has a definite truth value

1.1 Propositions and Logical Operations

- Sometimes simple statements are not enough (to express complicated ideas).
- Combine propositions to get compound propositions using certain composition rules called logical operations.

It is raining	It is cold	Proposition.

It is raining and it is cold	Compound Proposition.

- What determines the truth value of a compound proposition?
- Let's see some basic logical operations.

Conjunction Operator

Conjunction Operator: Represents logical "and" operation between two propositions.
The conjunction of propositions P and Q is conjunction of p and q is a new proposition, whose truth value is true when both P and Q are true, and false otherwise.

Notation:
$P \wedge Q$
Read as P and Q

Truth Table:

P	Q	$P \wedge Q$
True	True	True
True	False	False
False	True	False
False	False	False

Example:

If P represents "It is sunny"
and Q represents "I go to the beach,"
then $P \wedge Q$ represents "It is sunny and I go to the beach."

Disjunction Operator

Disjunction Operator: Represents logical "or" operation between two propositions.
The disjunction of propositions P and Q is a new proposition, whose truth value is true when either P or Q or both are true, and false otherwise.

Notation:
$P \vee Q$
Read as P or Q

Truth Table:

P	Q	$P \vee Q$
True	True	True
True	False	True
False	True	True
False	False	False

Example:

If P represents "I found a unicorn"
and Q represents "I won the lottery,"
then $P \vee Q$ represents "Either I found a unicorn, or I won the lottery"

Exclusive-Or Operator

Exclusive-Or Operator (XOR): Represents logical "exclusive-or" operation between two propositions.
The exclusive-or of propositions P and Q is a new proposition, whose truth value is true when either P or Q is true, but not both, and false when both P and Q are either true or false.

Notation:
$P \oplus Q$
Read as P exclusive-or Q

Truth Table:

P	Q	$P \oplus Q$
True	True	False
True	False	True
False	True	True
False	False	False

Example:

If P represents "I'm in the mood for pizza"
and Q represents "I'm in the mood for sushi,"
then $P \oplus Q$ represents "Either I'm in the mood for pizza or sushi, but not both!"

Negation Operator

Negation Operator ($\neg$): Represents the logical "not" operation on a proposition.
Given a proposition P, its negation $\neg P$ is a new proposition whose truth value is the opposite of P.

Truth Table:

Notation:
$\neg P$
Read as "not P"

P	$\neg P$
True	False
False	True

Example:

If P represents "It's a weekday," then $\neg P$ represents "It's not a weekday (i.e., it's the weekend)."

Logical Operator Problems

Problem 1: If "I find my keys" is represented by P, and "I'm late for work" is represented by Q, write the proposition "I'm not late for work and I found my keys."

Problem 2: Let P be "It's raining" and Q be "I forgot my umbrella." Write the proposition "It's not raining or I forgot my umbrella."

Problem 3: Suppose P is "I have cookies" and Q is "I have milk." Write the proposition "I have cookies and milk."

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Solutions

Problem 1: $\neg Q \wedge P$
Problem 2: $\neg P \vee Q$
Problem 3: $P \wedge Q$

1.1 Propositions and Logical Operations

Let p and q be propositions, then under what conditions

1. $p \oplus q \neq p \vee q$
2. $p \vee q = p \wedge q$
3. $p \oplus q = p \wedge q$

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3. $p \oplus q = p \wedge q$

Solutions:

1. $p = T, q = T$
2. $p = T, q = T$
3. $p = F, q = F$

Evaluating Compound Propositions

When evaluating compound propositions with logical operators, the order of operations is as follows:

1. Evaluate expressions inside parentheses.
2. Apply the negation ($\neg$) operator.
3. Apply the conjunction ($\wedge$) operator.
4. Apply the disjunction ($\vee$) operator.

Example

Evaluate $P \wedge \neg Q \vee R$ given $P =$ "Sunny", $Q =$ "Beach", $R =$ "Umbrella".

1. $\neg Q$ is "Not at Beach".
2. $P \wedge \neg Q$ is "Sunny and Not at Beach".
3. $P \wedge \neg Q \vee R$ is "Sunny and Not at Beach, or Umbrella".

Solving Compound Proposition

Example Proposition: $s = (p \vee q) \wedge \neg(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$

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Solution: The truth values of s for all possible truth values of p and q are shown in the table. Therefore, the simplified proposition is: $s = p \oplus q$.