Discrete Structures

Fight for Survival

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Nested Quantifiers

- If a predicate has more than one variable, each variable must be bound by a separate quantifier.
- A logical expression with more than one quantifier that bind different variables in the same predicate is said to have nested quantifiers.
- Example:

$$\exists x \forall y P(x, y)$$

Nested Quantifiers

Lets see which are bound and free variables here.

Statement	Bound variable	Free variable

Nested Quantifiers

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Statement	Bound variable	Free variable
$\forall x \exists y L(x, y)$		

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$\forall x \exists y L(x, y, z)$		

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Nested Quantifiers - Example

"For every machine, there is a supervisor that operates it."

- Identify variables
- Generate Predicates
- Statement with nested quantifiers

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- Machine: m
- Supervisor: s
- $Q(m, s)$: m is operated by s.

Nested Quantifiers - Example

m1 s1
m2 s2
m3 s3
m4 s4
m5

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- Generate Predicates
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- $Q(m, s)$: m is operated by s.

$$\forall m \exists s Q(m, s)$$

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m1 s1
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- Domain of m is $\{m_1, m_2, m_3, m_4, m_5\}$
- Domain of s is $\{s_1, s_2, s_3, s_4\}$

$$\forall m \exists s Q(m, s)$$

$$\equiv$$

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$$\forall m \exists s Q(m, s)$$

$$\equiv$$

$$\exists s Q(m_1, s) \wedge \exists s Q(m_2, s) \wedge \exists s Q(m_3, s) \wedge \exists s Q(m_4, s) \wedge \exists s Q(m_5, s)$$

Nested Quantifiers - Example

"There is a student with A's in all courses."

Student: s
Course: c
$G(s,c)$: s scored A in c

There exists some s for which $G(s,c)$ is true for all c.

$$\exists s \forall c G(s,c)$$

Nested Quantifiers - Example

"There is a student with A's in all courses."

Domain of s: $\{s_1, s_2, s_3\}$

Domain of c: $\{c_1, c_2, c_3\}$

$$\exists s \forall c G(s, c)$$

$$\equiv$$

$$(\forall c G(s_1, c)) \lor (\forall c G(s_2, c)) \lor (\forall c G(s_3, c)))$$

Nested Quantifiers Precedence and Operators

Precedence	Operators
1st	$\forall$ (Universal Quantification)
2nd	$\exists$ (Existential Quantification)
3rd	$\neg$ (Negation)
4th	$\wedge$ (Conjunction, AND)
5th	$\vee$ (Disjunction, OR)
6th	$\rightarrow$
7th	$\leftrightarrow$

- Precedence determines the order of evaluating nested quantifiers and operators in logical expressions.
- Operators with higher precedence are evaluated before those with lower precedence.

Nested Quantifiers and Precedence

Consider the following logical formula:

$$\forall x \neg \exists y p(x, y) \to \forall x q(x)$$

Evaluation Order:

1. The formula is equivalent to the following:

$$(\forall x(\neg(\exists y p(x, y)))) \to (\forall x q(x))$$

Nested Quantifiers: a summary

What does the following mean?

Domain of x: $\{x_1, x_2, x_3\}$

Domain of y: $\{y_1, y_2, y_3\}$

1.
2.
3.
4.
5.

	x_1	x_2	x_3	x_4
y_1				
y_2				
y_3				
y_4				
y_5				

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What does the following mean?

Domain of x: $\{x_1, x_2, x_3\}$

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5. when $\exists x \exists y P(x, y)$ is true.

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Nested Quantifiers: a summary

Statement	When True?	When False?
$\forall x \forall y P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

Predicate Logic

Now, lets try to write these statements using quantifiers.

$$F(x, y)\text{: } x \text{ can fool } y.$$

(The domain consists of all people in the world).

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- No one can fool himself/herself. $\neg \exists x F(x, x)$

Predicate Logic

Now, lets see what does these statement mean.

$$\exists x \exists y ((x^2 = y^2) \wedge (x \neq y))$$

[a]

[a] for real numbers.

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$$\exists x \exists y((P(x) \wedge P(y)) \wedge (x \neq y)) \wedge \forall z(P(z) \rightarrow (z = x) \vee (z = y))$$

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Now, lets see what does these statement mean.

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There exists two distinct real numbers whose squares are equal.

$$\exists x \exists y ((P(x) \land P(y)) \land (x \neq y)) \land \forall z (P(z) \rightarrow (z = x) \lor (z = y))$$

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There exists two distinct values that make the statement true
AND
Out of all the values, the statement is true for only x or y.

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Out of all the values, the statement is true for only x or y.

There are exactly two values that make the

Example:

I eat spinach (S) or ice cream (I). If I study logic (L) then I will pass the exam (E). If I eat ice cream, then I will study logic. If I eat spinach, then I will play golf (G). I failed the exam.

Predicate Logic and Proofs

Example:

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Therefore, I played Golf!

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Therefore, I played Golf!

Our goal is to verify this argument.[1]

[1] once we have understood it!

Predicate Logic and Proofs

Example:

1. I eat spinach (S) or ice cream (I). $S \vee I$
2. If I study logic (L), then I will pass the exam (E). $L \rightarrow E$
3. If I eat ice cream, then I will study logic (L). $I \rightarrow L$
4. If I eat spinach (S), then I will play golf (G). $S \rightarrow G$
5. I failed the exam. $\neg E$

Example:

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$$(S \vee I) \wedge (L \to E) \wedge (I \to L) \wedge (S \to G) \wedge \neg E \to G$$

Predicate Logic and Proofs

Example:

$(S \vee I) \wedge (L \to E) \wedge (I \to L) \wedge (S \to G) \wedge \neg E \to G$

- So, the goal is to simplify statements (arguments) like these and show if the conclusion holds or not.
- How can we simplify?
- By using rules of inference.
- Lets see some of them.

Rules of Inference

Rules of inference are the basic building blocks of logic. They allow us to derive new propositions from existing propositions, written in math form.

Rule	Premise(s)	Conclusion
Modus Ponens	$p \land (p \to q)$	q
Modus Tollens	$\neg q \land (p \to q)$	$\neg p$
Hypothetical Syllogism	$(p \to q) \land (q \to r)$	$p \to r$
Disjunctive syllogism	$(p \lor q) \land \neg p$	q

Table: Rules of Inference 1/2

Rules of inference

Rule	Premise(s)	Conclusion
Addition	p	$p \vee q$
Simplification	$p \wedge q$	p
Conjunction	$(p) \wedge (q)$	$p \wedge q$
Resolution	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$
Instantiation	$\forall x P(x)$	for arbitrary $cP(c)$
Generalization	for arbitrary $cP(c)$	$\forall x P(x)$

Table: Rules of Inference 2/2

Rules of Inference

State which rule of inference is the basis of the following argument: "It is below freezing and raining now. Therefore, it is below freezing now."

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"It is below freezing and raining now. Therefore, it is below freezing now."

Solution:

Let:
p be the proposition "It is below freezing now," and let q be the proposition "It is raining now."

This argument is of the form:

$p \wedge q$

$\therefore p$

This argument uses the simplification rule.

Example: Example: Prove that the argument with premises $A \vee C \to D, \neg B, A \vee B$ and with the conclusion D is valid.

What we're really being asked to do is Prove

$$(A \vee C \to D) \wedge \neg B \wedge (A \vee B) \to D$$

is true.

Example of a Proof

Prove that

$$(A \lor C \to D) \land \neg B \land (A \lor B) \to D$$

is true.

	Statements	Reason
1	$A \lor C \to D)$	Premise
2	$\neg B$	Premise
3	$A \lor B$	Premise
4		
5		
6		
7		

Example of a Proof

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3	$A \vee B$	Premise
4	A	
5		
6		
7		

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4	A	2, 3, Disjunctive Syll.
5		
6		
7		

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5	$A \lor C$	
6		
7		

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3	$A \lor B$	Premise
4	A	2, 3, Disjunctive Syll.
5	$A \lor C$	4, Addition
6		
7		

Example of a Proof

Prove that

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6	D	1,5, Modus Ponen
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5	$A \vee C$	4, Addition
6	D	1,5, Modus Ponen
7	QED	1-6

A Practice Problem:

Prove that the argument with **Premises :** $(\neg p \land q), (r \rightarrow p), (s \rightarrow t), (\neg r \rightarrow s)$, and Conclusion: t
is a valid argument.

Our goal is to prove:
Premises : $A \to B$

- So far, we have seen how to use inference rules and show that hypotheses on L.H.S imply the conclusion on the R.H.S.
- There is an another interesting way - Indirect proofs.
- First recall two facts:

Indirect Proofs

Our goal is to prove:
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1. A proposition cannot be true and false at the same time. $(A \wedge \neg A) = False$ (a contradiction).

Indirect Proofs

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- First recall two facts:

1. A proposition cannot be true and false at the same time. $(A \wedge \neg A) = False$ (a contradiction).

2. If $(A \rightarrow B)$ then $(\neg B \rightarrow \neg A)$. Recall modus tollens. In words, if A is true, we know B is true. B is necessary for A. Consequently, if B is false, A must be false. Hence, $(\neg B \rightarrow \neg A)$.

Indirect Proofs

Show that if 3n+2 is odd, then n is odd. (n is an integer) P: 3n+2 is odd
Q: n is odd
We need to show: $P \to Q$

Lets try a direct approach first.

- $3n + 2$ is odd. Premise
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There does not seem to be a direct way to conclude from here that n is odd. Lets try our new approach.

Indirect Proofs: Proof by Contrapositive

P: 3n+2 is odd
Q: n is odd
We need to show: $P \to Q$

Lets try a different approach.

1. $3n+2$ is odd. Premise
2.
3.
4.
5.
6.
7.
8.

Indirect Proofs: Proof by Contrapositive

P: 3n+2 is odd
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Lets try a different approach.

1. $3n + 2$ is odd. Premise
2. $\neg Q$ (n is even). Assumption
3.
4.
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Lets try a different approach.

1. $3n+2$ is odd. Premise
2. $\neg Q$ (n is even). Assumption
3. $n = 2k$ By the definition of even numbers
4. $3n+2 = 3(2k)+2$ Replacing n in (3n+2)
5.
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Indirect Proofs: Proof by Contrapositive

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4. $3n+2 = 3(2k)+2$ Replacing n in (3n+2)
5. $2(3k+1)$ Simplifying line 3
6.
7.
8.

Indirect Proofs: Proof by Contrapositive

P: 3n+2 is odd
Q: n is odd
We need to show: $P \rightarrow Q$

Lets try a different approach.

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2. $\neg Q$ (n is even). — Assumption
3. $n = 2k$ — By the definition of even numbers
4. $3n+2 = 3(2k)+2$ — Replacing n in (3n+2)
5. $2(3k+1)$ — Simplifying line 3
6. $2(3k+1)$ is even — By the definition of even numbers
7. $\neg P$ — From line 5
8. $P \rightarrow Q$ — QED.

Example of a Proof

	Statements	Reason
1	$S \lor I$	Premise
2	$L \to E$	Premise
3	$I \to L$	Premise
4	$S \to G$	Premise
5	$\neg E$	Premise
6		
7		
8		
9		
10		

Example of a Proof

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1	$S \vee I$	Premise
2	$L \rightarrow E$	Premise
3	$I \rightarrow L$	Premise
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5	$\neg E$	Premise
6	$\neg L$	
7		
8		
9		
10		

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6	$\neg L$	2,5,Modus tollens
7		
8		
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7	$\neg I$	
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7	$\neg I$	3,6, Modus tollens
8	S	
9		
10		

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5	$\neg E$	Premise
6	$\neg L$	2,5,Modus tollens
7	$\neg I$	3,6, Modus tollens
8	S	1,7, Disjunctive Syllogism
9		
10		

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5	$\neg E$	Premise
6	$\neg L$	2,5,Modus tollens
7	$\neg I$	3,6, Modus tollens
8	S	1,7, Disjunctive Syllogism
9	G	
10		

Example of a Proof

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2	$L \to E$	Premise
3	$I \to L$	Premise
4	$S \to G$	Premise
5	$\neg E$	Premise
6	$\neg L$	2,5,Modus tollens
7	$\neg I$	3,6, Modus tollens
8	S	1,7, Disjunctive Syllogism
9	G	4,8, Modus Ponens
10	QED	1-9