Discrete Structures

Fight for Survival

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Limitations of Propositional Logic

Propositional Logic Limitations

- Inadequate for handling statements involving many objects.
- Each statement must be repeated for each object.

Example

"If Alice is a CS graduate, then she knows logic."
"If Bob is a CS graduate, then he knows logic."
"If Charlie is a CS graduate, then he knows logic."
Too many statements!

Escape the limitations with Predicate Logic!

Predicate Logic: Beyond Propositional Logic

- Explicitly models objects and their properties.
- Allows statements with variables and quantification.

Basic Building Blocks

- Constants: Specific objects like "Mars," "2023," or "Mona Lisa."
- Variables: Represent objects of specific types, defined by the universe of discourse (e.g., planets, years, artworks).
- Predicates: Express properties or relations among objects, can involve one, two, or many variables/constants.

- Constant: "Earth," "2023," "The Mona Lisa."
- Variable: x, y (depending on the universe).
- Predicate: Orbits(x, y), IsLeapYear(x), Painted($Leonardo$, $MonaLisa$).

Predicate Truth Values

Consider a predicate $P(x)$ that represents the statement:

$P(x)$

x is a prime number

This is known as a propositional function!

What are the truth values of the following propositions?

- $P(2)$
- $P(3)$
- $P(4)$
- $P(5)$
- $P(6)$
- $P(7)$

All statements $P(2)$, $P(3)$, $P(4)$, $P(5)$, $P(6)$, $P(7)$ are propositions.

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Predicate Logic Representation

Let's represent the statement: "Students missed breakfast."

We can use a predicate $M(x)$ to denote that x missed breakfast, where x represents a student.

$M(x)$

x missed breakfast

In this representation:

- x is a variable representing a student.
- $M(x)$ is a predicate indicating that x missed breakfast.

This way, we can express the statement "Students missed breakfast" in predicate logic.

Example

Let Q(x,y,z) denotes the statement "$x^2 + y^2 = z^2$".

- What is the truth value of Q(3,4,5)?
- What is the truth value of Q(2,2,3)?
- How many values of (x,y,z) make the predicate true?

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Let Q(x,y,z) denotes the statement "$x^2 + y^2 = z^2$".

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Solution,
Since $3^2 + 4^2 = 5^2 = 25$ where Q(3,4,5) is True.
Since $2^2 + 2^2 = 8 \neq 3^2 = 9$ where Q(2,2,3) is False.
There are infinitely many values for which Q(x,y,z) is true.
(Hint: How many right angle triangles are there?)