Discrete Structures

Fight for Survival

Dr. Mudassir Shabbir

August 27, 2026

Graph Representations

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A representation is needed to:

- -store graphs in memory
- -share graphs information

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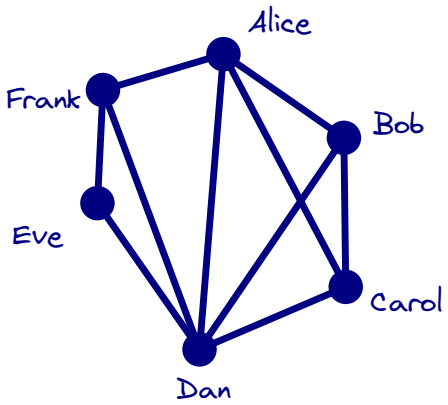

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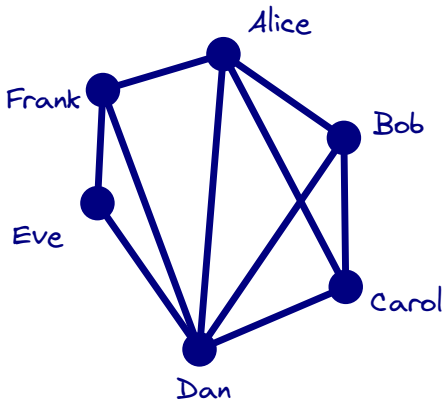

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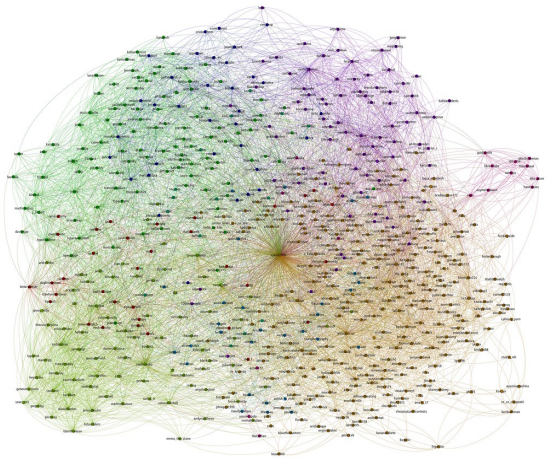

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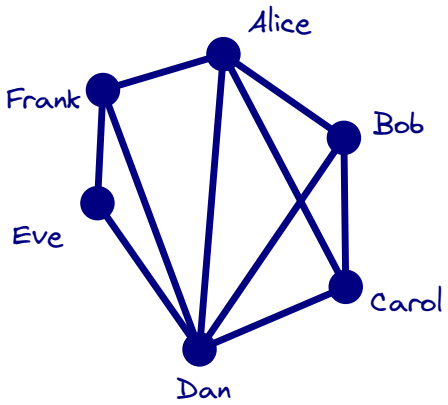

Three common ways to represent:

1. list of all edges
2. separate lists of edges for each vertex
3. yes/no for each possible edge

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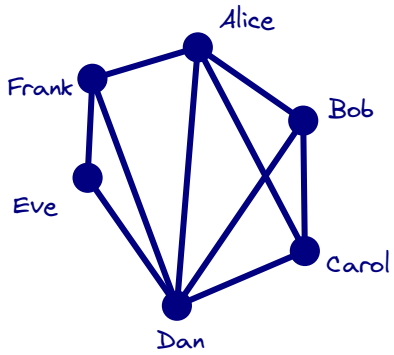
1. Edge List



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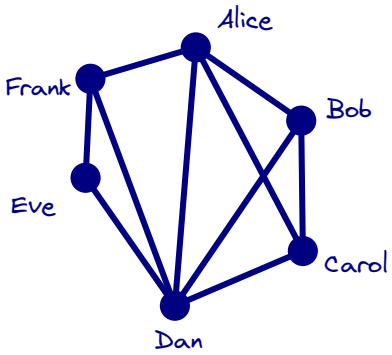

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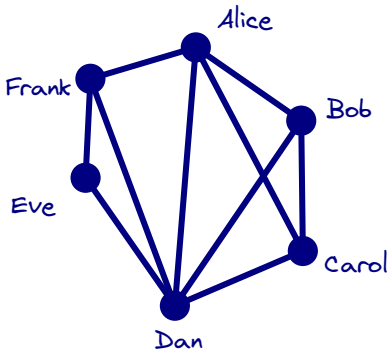

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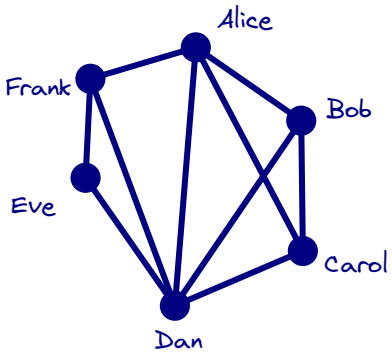

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Need to go through the whole list to check wheher
Dan and Frank are friends?
OR
even to check if Eve has any friends?

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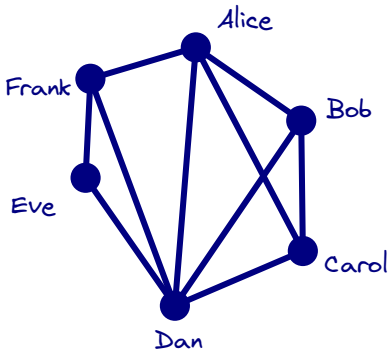

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Commonly used in Machine Learning algorithms that deal with graphs with trillions of edges.

Graph Representations

A representation is needed to:

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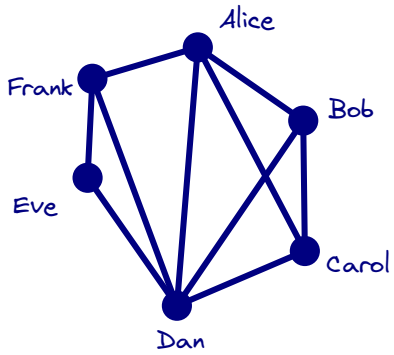
2. Adjacency List



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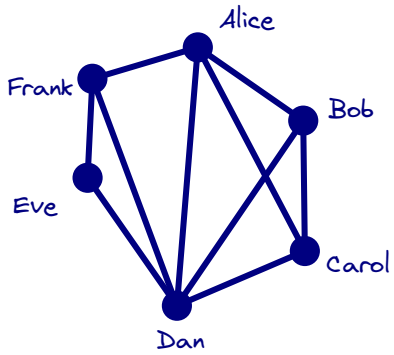

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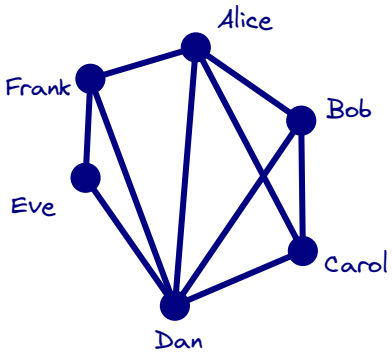

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Dan -> Alice, Bob, Carol, Eve, Frank
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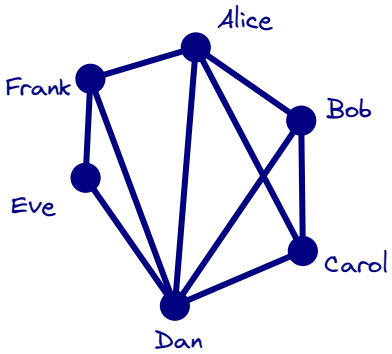

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Graph Representations

A representation is needed to:

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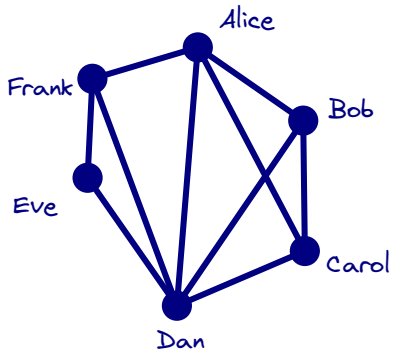
3. Adjacency Matrix



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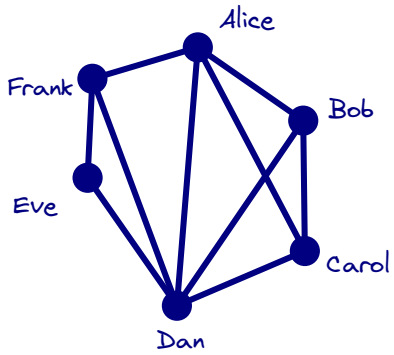
IDEA: for each possible edge answer YES/NO.



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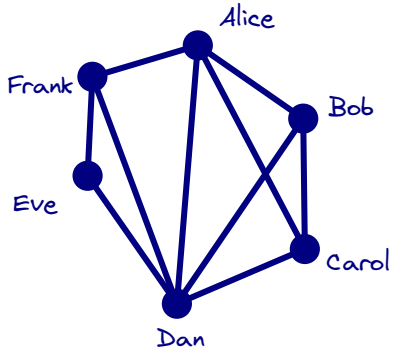

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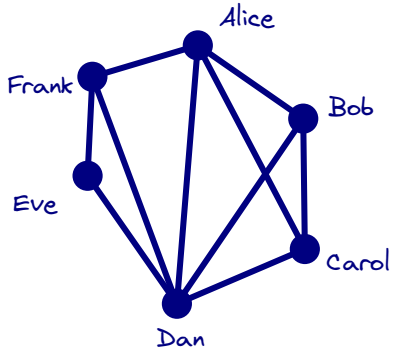

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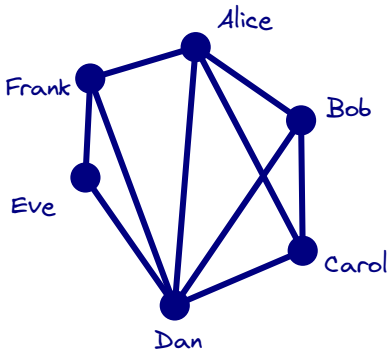

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B						
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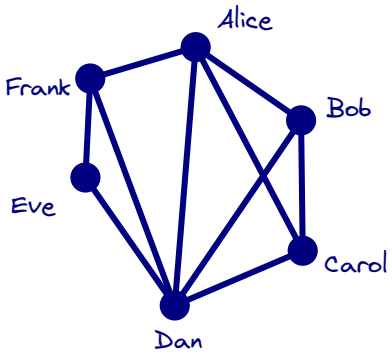

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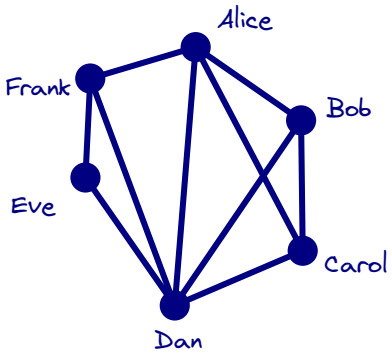

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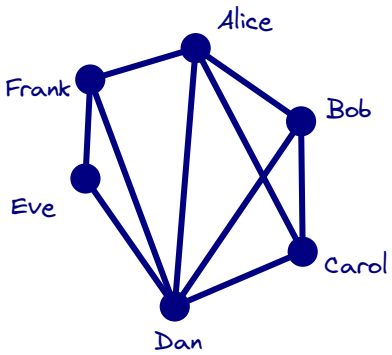

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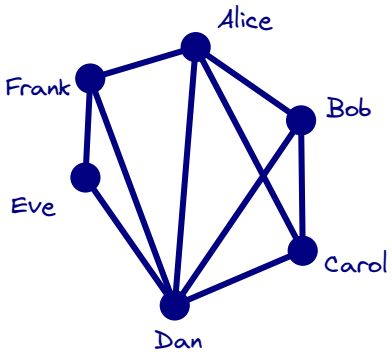

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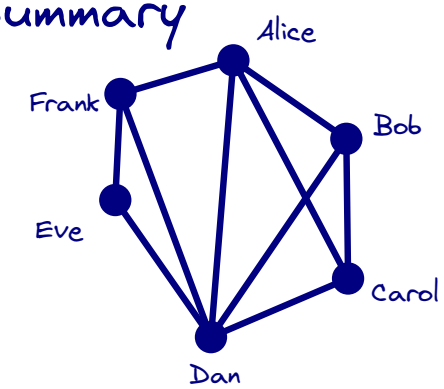

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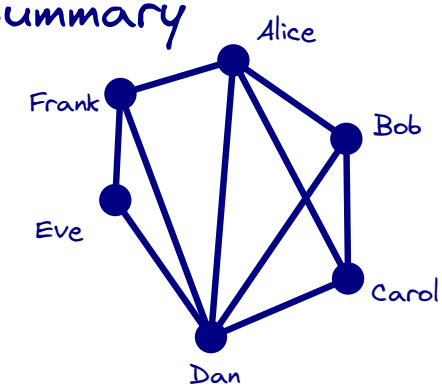

1. Edge List

Memory footprint:

$\Theta(|E|)$ we keep all the edges.

2. Adjacency List

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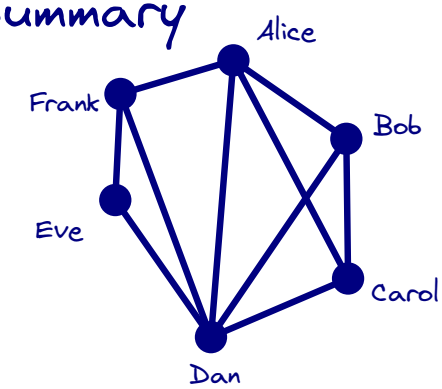

1. Edge List

Memory footprint:

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2. Adjacency List

Memory footprint:

$\Theta(|V| + |E|)$ We keep neighbors for each vertex.

3. Adjacency Matrix

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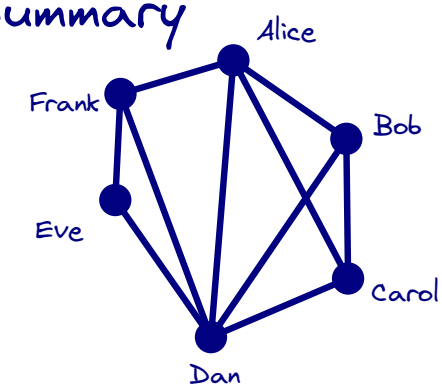

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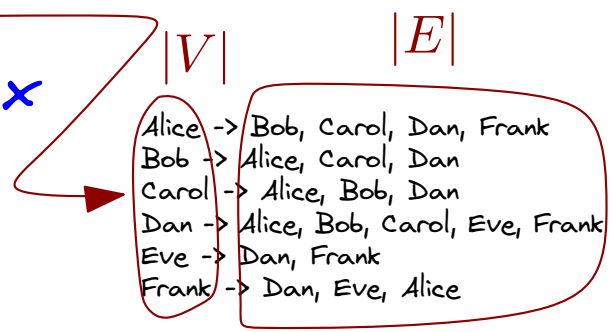

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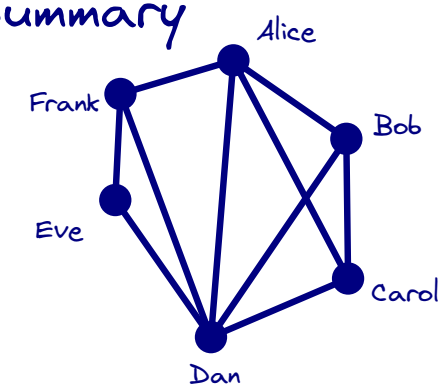

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Memory footprint:

$\Theta(|E|)$ we keep all the edges.

2. Adjacency List

$$\Theta(|V|) \text{ if } |V| > |E|$$
$$\Theta(|E|) \text{ if } |V| < |E|$$
$$= \max(|V|, |E|)$$

Memory footprint:

$\Theta(|V| + |E|)$ We keep neighbors for each vertex.

3. Adjacency Matrix

$|V|$ $|E|$

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TOPHAT Question

- For which graphs, adjacency list takes $\Theta(|E|)$ memory?

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- For which graphs, adjacency list takes $\Theta(|E|)$ memory?
- All "connected graphs" because they have $\Omega(|V|)$ edges.

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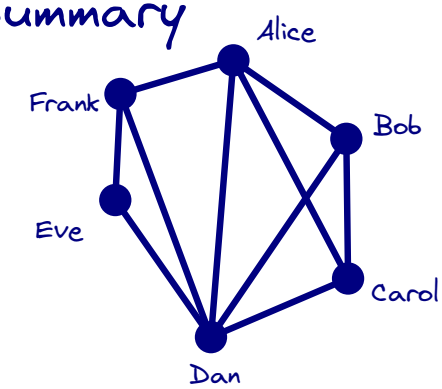

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Memory footprint:

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3. Adjacency Matrix

Memory footprint:

$\Theta(|V|^2)$ We keep a matrix of all possible edges.

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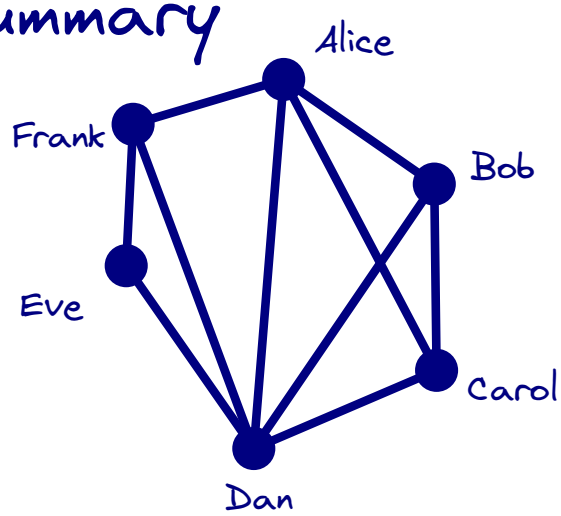
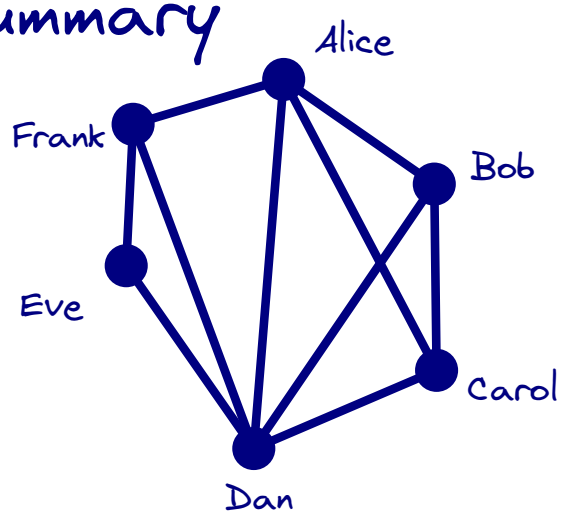


Graph Representations: Summary

1. Edge List



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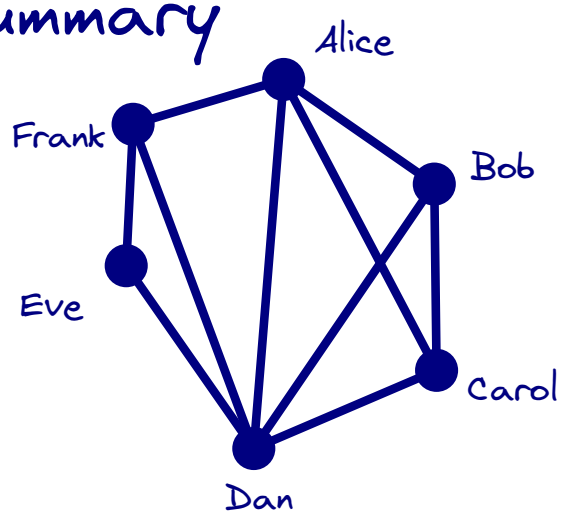

List all neighbors of a vertex:

$\Theta(|E|)$ we have to go through all the edges.

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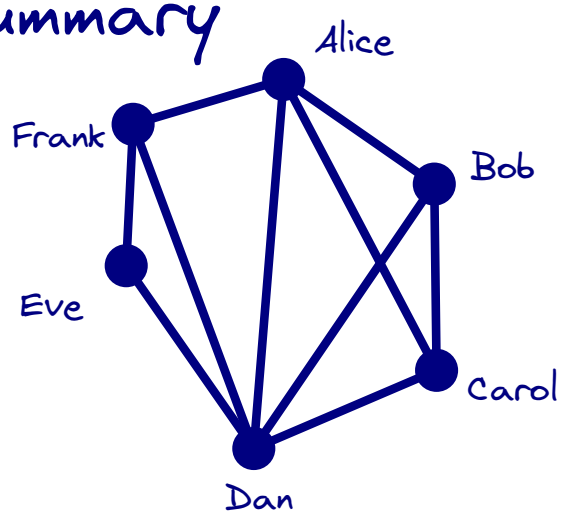

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2. Adjacency List

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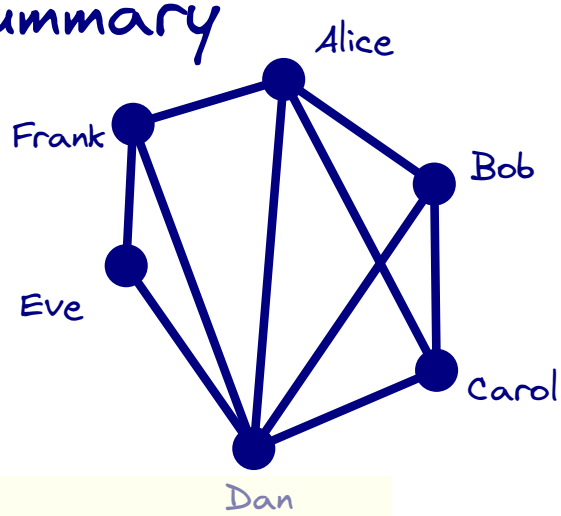


1. Edge List

List all neighbors of a vertex:

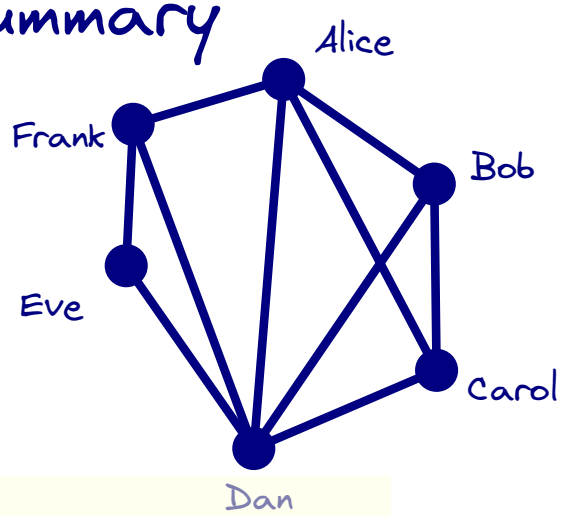
$\Theta(|E|)$ we have to go through all the edges.

2. Adjacency List

List all neighbors of a vertex:

$\Theta(|V|)$ Only need to check one of the lists.

Graph Representations: Summary



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List all neighbors of a vertex:

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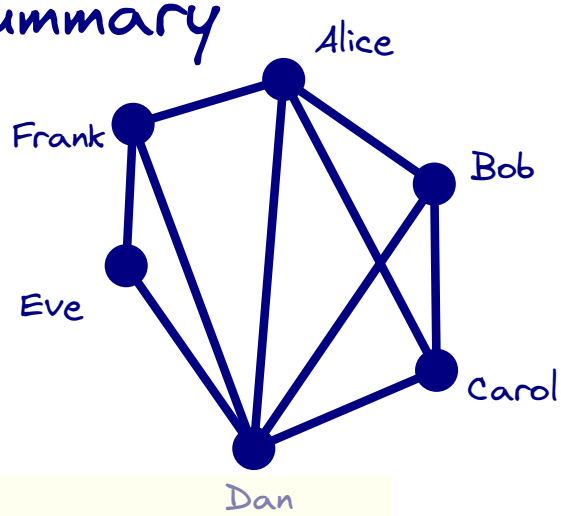
2. Adjacency List

List all neighbors of a vertex:

$\Theta(|V|)$ Only need to check one of the lists.

3. Adjacency Matrix

Graph Representations: Summary



1. Edge List

List all neighbors of a vertex:

$\Theta(|E|)$ we have to go through all the edges.

2. Adjacency List

List all neighbors of a vertex:

$\Theta(|V|)$ Only need to check one of the lists.

3. Adjacency Matrix

List all neighbors of a vertex:

$\Theta(|V|)$ Only need to check one row in the matrix.

Preferred Data Structures/Representations

- Traverse a graph?

Preferred Data Structures/Representations

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- Operations on a graph with 10^6 vertices?

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Preferred Data Structures/Representations

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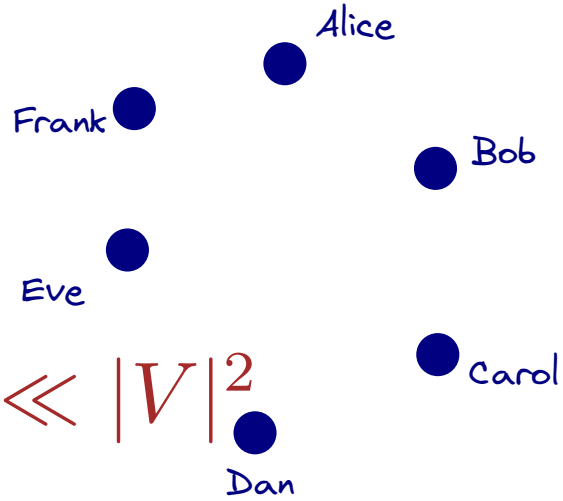
- Adjacency Matrix
- Count Neighbors of a vertex?
- Adjacency List
- Operations on a graph with 10^6 vertices?
- Adjacency List
- Operations on a sparse graph vs dense graph?

Sparse vs Dense Graphs

Sparse vs Dense Graphs

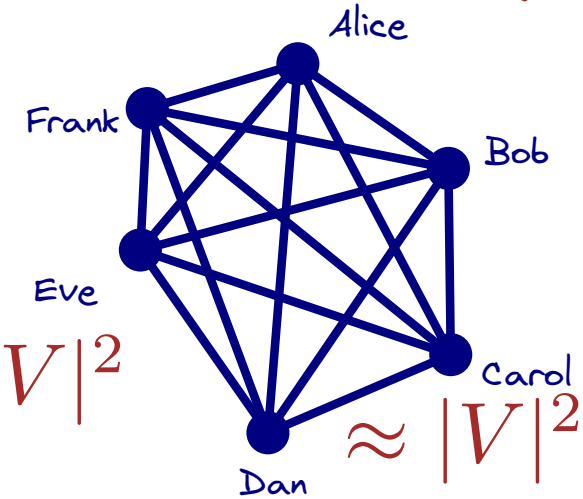
min. number of edges max. number of edges



0

$\ll |V|^2$

$\binom{|V|}{2} \approx |V|^2$



$\approx |V|^2$

SPARSE

twitter: 450 million vertices with 40-50 followers avg.

Facebook: 2.9 billion vertices with 190 friends avg.

DENSE

crypto-currency graph is complete.