Discrete Structures

Fight for Survival

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Sequences

Sequence: A function in which the domain is a subset of integers. In other words, a sequence is an ordered list of numbers. It can be finite or infinite.

Index k	0	1	2	3	4	5	6	7	8	9	10
Term g(k)	6	13	23	33	54	83	118	156	210	282	350

A sequence can "usually" be specified by an explicit formula showing how the value of term a_k depends on k.

For example, $a_k = 2^k, k \geq 0$ is an exponential sequence.

Sequences

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Arithmetic Sequences

A sequence of real numbers where each term after the initial term is found by taking the previous term and adding a fixed number called the common difference.

An arithmetic sequence (progression) is a sequence of form
$a, a+d, a+2d, \ldots, a+nd, \ldots$
where the initial term a and the common difference d are real numbers.

- Note that $a_k = a_0 + kd$
- Observe a "linear" growth.

Example:

An arithmetic sequence with $a = 3$ and $d = 2$:
$3, 5, 7, 9, 11, \ldots$

Arithmetic Sequences: Examples

A batsman scores a zero in his first match innings. He scores exactly five runs more than the previous game in every other innings. How much does he score in his 25^{th} match? What would be his batting average after 25 games?

Geometric Sequences

A sequence where each term after the initial term is found by taking the previous term and multiplying by a fixed number called the common ratio.

A geometric sequence (progression) is a sequence of form
$a, ar, ar^2, , ..., ar^n, ...$
where the initial term a and the common ratio r are real numbers.

- Here, $\frac{a_k}{a_{k-1}} = r$, for all k.
- Note that $a_k = r^k a_0$

An geometric progression is a discrete analogue of the exponential function $f(x) = ar^x$.

Example:

A geometric sequence with $a = 2$ and $r = 3:$

$2, 6, 18, 54, 162, \ldots$

Harmonic Sequences

Harmonic sequence is defined as:

A harmonic sequence (progression) is a sequence in which the reciprocals of the terms form an arithmetic sequence.

Example:

A harmonic sequence:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$$

Series

A series is the sum of the terms of a sequence.

Example: The sum of the first n natural numbers

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \sum_{k=1}^{n} k = \frac{n(n-1)}{2}$$

Arithmetic Series

An arithmetic series is the sum of the terms of an arithmetic sequence.
The sum of the first $n+1$ terms of a sequence, $a_0, a_2, \ldots, a_n$,is given by

$$S_n = \frac{(n+1)(a_0 + a_n)}{2}$$

Arithmetic Series

Geometric Series

A geometric series is the sum of the terms of a geometric sequence.

$$\sum_{k=0}^{n-1} ar^k = \frac{a}{1-r} \times (1 - r^n)$$

for $r \neq 1$.

Harmonic Series

A harmonic series is the sum of the terms of a harmonic sequence.
Example:

The sum of the first n terms of the harmonic series:

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\ldots+\frac{1}{n}=\sum_{k=1}^{n}\frac{1}{k}$$

Formulas

- Sum of an arithmetic series:

$$\sum_{k=1}^{n} a_k = \frac{n}{2}(2a_1 + (n-1)d)$$

- Sum of a finite geometric series:

$$\sum_{k=0}^{n-1} ar^k = a\frac{1-r^n}{1-r}$$

- Sum of a harmonic series:

$$\sum_{k=1}^{n} \frac{1}{k} = \gamma + \ln n$$

where $\gamma \approx 0.577$

Conclusion

- Sequences are ordered lists of numbers.
- Arithmetic sequences have a constant difference between terms.
- Geometric sequences have a constant ratio between terms.
- Harmonic sequences have reciprocals that form an arithmetic sequence.
- Series are the sum of the terms of a sequence.
- Arithmetic, geometric, and harmonic series have their own sum formulas.